Problems 2:
Binomial pricing

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Question 1

Let S be the price of stock at t, and suppose that at $t + \Delta t$ the stock can only change into two values: up $Su > S$ or down $Sd < S$ (here u and d represent the relative increase or decrease of the stock price). What are the probabilities of the up and down moves of the stock price, if the market is assumed to have no arbitrage opportunities?

Question 2

Let $S = £100$ be the stock price, which after time T can only change up $Su = £120$ or down $Sd = £80$. Compute the risk-neutral probabilities of up move Su and down move, assuming that the risk-free interest rate $r = 0$. How will these probabilities change, if $r > 0$?

Question 3

Assuming one step binomial model, consider stock with spot price $S = \$80$, price after up move $Su = \$100$ and after down move $Sd = \$60$. Assuming riskless rate of return $r = 0$, price European put option with the strike price $K = \$90$ and expiring in $T = 1$ year. Describe how the value changes as a function of the strike price. What changes if $r > 0$? What if the stock pays dividents with $s > 0$ APR?

Question 4

Assuming one step binomial model and risk-free rate $r = 0$, price European call option on stock described in the previous question with the strike price $K = \$90$ and expiring in $T = 1/2$ years.

Question 5

Consider a replicating portfolio $V = SX + B$ for a stock assumed to follow a one step binomial model (i.e. at the expiration date T the stock price S can only move up to Su or down to Sd). Assuming no arbitrage, derive the value X of the stock quantity and B of a riskless investment. Derive the value V of an option in the replicating portfolio.

Question 6

The one step Binomial pricing of an option is $V(t) = e^{-r(T-t)}[V_u\lambda + V_d(1-\lambda)]$. Derive the correspondong pricing formula, if the stock price can change twice during the period $T - t$ (i.e. the $N = 2$ step binomial model).

Question 7

Assume a $N = 2$ step binomial model for stock with spot price $S = £400$, one-step up price $Su = £550$ and down price $Sd = £350$. Price European call option with the strike price $K = £400$ expiring in $T = 1$ year, and assuming risk-free rate $r = 0$. What is the price of the corresponding put (i.e. with the same strie price K)?

Question 8

Assume a $N = 2$ step binomial model for stock with spot price $S = £5$, one-step up price $Su = £8$ and down price $Sd = £4$. Price European put option with the strike price $K = £6$ expiring in $T = 1$ year, and assuming risk-free rate $r = 0$. What is the price of the corresponding call (i.e. with the same strie price K)?

Question 9

BTC traded at $S = £2,000$, and put options on BTC with strike price $K = £1,500$ expiring in $T = 1$ year are traded at $£100$ per put. Suppose that after $T = 1$ the price drops to $S(T) = £1,000$. Compute and compare profits and returns on three investments: 1) one BTC; 2) equivalent amount invested in puts; 3) the amount split equally between BTC and puts. What are the profits and returns, if the price increases to $S(T) = £4,000$? What are the expected profits and expected returns on the three investments? How can these expected values be estimated if $S(T) \in \{£1,000, £4,000\}$?