Problems 6:
Stochastic differential equations and integrals

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Question 1

Is the process $x(t)$, described by a stochastic differential equation $dx(t) = x(t)[\mu\, dt + \sigma\, dw(t)]$, differentiable at any t? Is the process $y(t) = \ln(x(t))$ differentiable?

Answer: *No, the process $x(t)$ is nowhere differentiable, because dx contains stochastic differential dw (white noise).*

Question 2

Consider the following stochastic differential equations:

a) $dx = \mu\, dt + \sigma\, dw$

b) $dx = \mu x\, dt + \sigma\, dw$

c) $dx = x(\mu\, dt + \sigma\, dw)$

d) $dx = \sin(x)\, dt + \cos(x)\, dw$

What are the drift $f(x,t)$ and diffusion $g(x,t)$ parts in each of these equations?

Answer:

a) $f(x,t) = \mu$, $g(x,t) = \sigma$

b) $f(x,t) = \mu x(t)$, $g(x,t) = \sigma$

c) $f(x,t) = \mu x(t)$, $g(x,t) = \sigma x(t)$

d) $f(x.t) = \sin(x(t))$, $g(x,t) = \cos(x(t))$